PROPAGATION OF SMALL PERTURBATIONS AND THE SLIP LINE IN A PLASTIC MEDIUM

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A study is made of surfaces, describing the propagation of perturbations in a plastic medium, described by equations proposed in [1]. A review of work in which these processes are investigated on the basis of other models can be found in [2]. A partial case of the equations discussed here was proposed in [3]. The surfaces of the propagation of the waves are described using an acoustical matrix, which determines three waves: quasilongitudinal and two quasitransverse. The acoustical matrix is symmetrical and positively defined; these properties are determined by the required correctness (hyperbolicity) of the system of differential equations under consideration. The communication [4] is devoted to a description of a class of hyperbolic systems similar to that considered here. It is found that the acoustical matrix corresponding to the system of differential equations under consideration here, for an elasticoplastic medium, can degenerate at several surfaces. This kind of degeneration corresponds to the reversion to zero of the velocity of one of the quasitransverse waves. In the plane case of a system of equations linearized in some manner, these degenerate surfaces coincide with the slip surfaces of the classical theory of plasticity.

The dynamic equations of an isotropic elasticoplastic medium, in the rectangular Cartesian system of coordinates x_i proposed in [1], have the form

$$\rho du_i/dt - \partial\sigma_{ij}/\partial x_j = 0, \; dh_{ij}/dt - U_{i\alpha}q_{\alpha\beta}U_{j\beta} = 0, \quad (1)$$
$$\rho E_s dS/dt - L\sigma_{ij}\partial u_i/\partial x_j + (l_\alpha\sigma_\alpha)\partial u_\beta/\partial x_\beta = 0,$$

where $d/dt = \partial/\partial t + u_\alpha \partial/\partial x_\alpha$; u_i is the vector of the velocity; σ_{ij} is the tensor of the stresses; h_{ij} is the tensor of the effective elastic Hencky deformations; S is the entropy; ρ is the density. The tensor U_{ij} forms an orthogonal matrix U, relating σ_{ij} and h_{ij} to the principal axes:

$$\|\sigma_{ij}\| = U\begin{pmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{pmatrix}U^*, \; \|h_{ij}\| = U\begin{pmatrix}h_1 & 0 & 0\\ 0 & h_2 & 0\\ 0 & 0 & h_3\end{pmatrix}U^*, \quad UU^* = I.$$

The stresses σ_{ij} are connected with the effective elastic deformations h_{ij} by the formulas

$$\sigma_{ij} = \rho\partial E/\partial h_{ij}, \; \rho = \rho_0 \exp(-h_{11}-h_{22}-h_{33}),$$

or, at the principal axes

$$\sigma_i = \rho\partial E/\partial h_i, \; \rho = \rho_0 \exp(-h_1-h_2-h_3),$$

$E = E(h_1, h_2, h_3, S)$ is the density of the internal energy (the equation of state). The temperature is calculated using the formula $T = E_S$. The values of q_{ij} are calculated using the formulas

$$q_{ii} = (1-L)\omega_{ii} + l_i(\omega_{11}+\omega_{22}+\omega_{33}),$$
$$q_{ij} = -\frac{h_i - h_j}{e^{-2h_i} - e^{-2h_j}}\left(e^{-2h_i}\omega_{ij} + e^{-2h_j}\omega_{ji}\right), \quad i \neq j,$$

where $\omega_{ij} = U_{\alpha i}\dfrac{\partial u_\alpha}{\partial x_\beta}U_{\beta j}$.

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The parameter of the plasticity L in the plastic region can depend on all the invariants of the stresses and on the temperature. The elastic region is separated out in the following manner: L = 0, if

$$h=\frac{1}{\sqrt{2}}[(h_1-h_2)^2+(h_2-h_3)^2+(h_3-h_1)^2]^{1/2}<h_*=\text{const},$$

or

$$(L\sigma_1-l_\alpha\sigma_\alpha)\frac{dh_1}{dt}+(L\sigma_2-l_\alpha\sigma_\alpha)\frac{dh_2}{dt}+(L\sigma_3-l_\alpha\sigma_\alpha)\frac{dh_3}{dt}<0.$$

The values of l_i are expressed in terms of L and the equation of state. For an equation of state of the form

$$E(h_1,h_2,h_3,S)=E^0(\rho,S)+B(\rho)\sum_{i=1}^{3}\left(h_i-\frac{h_1+h_2+h_3}{3}\right)^2$$

we have

$$l_i=\frac{L}{3}\left[1-\frac{3}{2B}\left(\frac{T_\rho}{T}-\frac{B_\rho}{B}\right)\left(\sigma_i-\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)\right].$$

The propagation of the waves of small perturbations is described using characteristic surfaces. The equation of the normal to the characteristics in a system of equations connected with the principal axes of the stresses for the system (1) is written in [1]. If we designate by (τ, ξ_1, ξ_2, ξ_3) the vector of the normal to the characteristic surface, then, the equation of the characteristic of the normals, corresponding to the propagation of acoustical waves, has the form

$$\det(\Omega^2 I-\Lambda)=0,$$

where $\Omega=\tau+u_\alpha\xi_\alpha$; Λ is the acoustical matrix:

$$\Lambda=\begin{pmatrix} L_1\xi_1^2+M_3e^{2h_2}\xi_2^2+M_2e^{2h_3}\xi_3^2 & P_3\xi_1\xi_2 & P_2\xi_1\xi_3\\ P_3\xi_2\xi_1 & M_3e^{2h_1}\xi_1^2+L_2\xi_2^2+M_1e^{2h_3}\xi_3^2 & P_1\xi_2\xi_3\\ P_2\xi_3\xi_1 & P_1\xi_3\xi_2 & M_2e^{2h_1}\xi_1^2+M_1e^{2h_2}\xi_2^2+L_3\xi_3^2\end{pmatrix}.$$

The modules of L_i, M_i, P_i are expressed in terms of derivatives of the equation of state and the parameter of the plasticity L.

It is obvious that, if we write the acoustical matrix in the case where the principal axes of the stresses do not coincide with the coordinate axes, then, its structure becomes complicated, as well as the formulas for calculation of the modules, into which the elements of the matrix of the rotation U will now enter.

We note that, if the vector of the normal to the characteristic surface (τ, ξ_1, ξ_2, ξ_3) is known, then, the characteristic surface (the surface of the propagation of the waves of small perturbations) $\Phi(t_1, x_1, x_2, x_3)$ = const is determined by the equation

$$\tau\partial\Phi/\partial t+\xi_i\partial\Phi/\partial x_i=0.$$

The acoustical matrix Λ for different types of media is assumed to be positively defined. This occurs, e.g., for all anisotropic elastic media, and means that small perturbations are propagated with nonzero velocities.

For the equations under discussion here, it is found that, at some surfaces $\Sigma(\xi_1, \xi_2, \xi_3)$ = const, the matrix Λ may degenerate, i.e., there is an eigenvalue $\Omega^2 = 0$. This means that, at some surface $\Psi(x_1, x_2, x_3)$ = const, the velocity of one quasitransverse wave (a shear wave) is equal to zero.

As an illustration of this, let us consider a two-dimensional unsteady-state system of equations, obtained from (1) by some linearization. We postulate that all the sought functions depend on two spatial variables, $x = x_1$ and $y = x_2$. We shall consider the processes without taking account of temperature effects, excluding the equation for the entropy and the dependence of the equation of state on the entropy ($E = E(h_1, h_2, h_3)$). We assume that the principal values of h_i of the effective elastic deformations are small, and thus,

$$q_{ij} = (1/2)(\omega_{ij} + \omega_{ji}),\ i \neq j.$$

We assume also that the velocities and their gradients are small, so that expressions of the form of $u_\alpha \partial u_k/\partial x_\alpha$ can be discarded. We represent the matrix U in the form

$$U = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix},$$

here

$$\omega_{11} = \cos^2\varphi \partial u/\partial x - \sin\varphi\cos\varphi\,(\partial v/\partial x + \partial u/\partial y) + \sin^2\varphi \partial v/\partial y,$$
$$\omega_{22} = \sin^2\varphi \partial u/\partial x + \sin\varphi\cos\varphi(\partial v/\partial x + \partial u/\partial y) + \cos^2\varphi \partial v/\partial y,$$
$$\omega_{12} = \sin\varphi\cos\varphi(\partial u/\partial x - \partial v/\partial y) + \cos^2\varphi \partial u/\partial y - \sin^2\varphi \partial v/\partial x,$$
$$\omega_{21} = \sin\varphi\cos\varphi(\partial u/\partial x - \partial v/\partial y) - \sin^2\varphi \partial u/\partial y + \cos^2\varphi \partial v/\partial x.$$

The connection between the stresses σ_{ij} and the effective elastic deformations h_{ij} is given in the form of Hooke's law: $\sigma_{ij} = \lambda(h_{11} + h_{22} + h_{33})\delta_{ij} + 2\mu h_{ij}$. In this case, $l_1 = l_2 = l_3 = L/3$. After the simplifications made, the system (1) assumes the form

$$\begin{gathered}
\rho_0 \partial u/\partial t = \partial\sigma_{11}/\partial x + \partial\sigma_{12}/\partial y,\ \rho_0 \partial v/\partial t = \partial\sigma_{21}/\partial x + \partial\sigma_{22}/\partial y, \\
\frac{\partial h_{11}}{\partial t} = (1-L)\left[\left(1 - \frac{1}{2}\sin^2 2\varphi\right)\frac{\partial u}{\partial x} + \frac{1}{2}\sin^2 2\varphi\frac{\partial v}{\partial y} - \frac{1}{2}\sin 2\varphi\cos 2\varphi\right. \\
\left.\times\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{1}{3}L\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{1}{2}\sin^2 2\varphi\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + \frac{1}{2}\sin 2\varphi\cos 2\varphi\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \\
\frac{\partial h_{22}}{dt} = (1-L)\left[\frac{1}{2}\sin^2 2\varphi\frac{\partial u}{\partial x} + \left(1 - \frac{1}{2}\sin^2 2\varphi\right)\frac{\partial v}{\partial y} + \frac{1}{2}\sin 2\varphi\cos 2\varphi\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] \\
+ \frac{1}{3}L\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \frac{1}{2}\sin^2 2\varphi\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) - \frac{1}{2}\sin 2\varphi\cos 2\varphi\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \\
\frac{\partial h_{33}}{\partial t} = \frac{1}{3}L\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \\
\frac{\partial h_{12}}{\partial t} = (1-L)\left[\frac{1}{2}\sin^2 2\varphi\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) - \frac{1}{2}\sin 2\varphi\cos 2\varphi\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right] \\
+ \frac{1}{2}\sin 2\varphi\cos 2\varphi\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + \frac{1}{2}\cos^2 2\varphi\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right),
\end{gathered} \tag{2}$$

where $\sigma_{ij} = \lambda(h_{11} + h_{22} + h_{33})\delta_{ij} + 2\mu h_{ij}$.

It can be shown that if, in the equations for h_{ij} we set L = 0, then, we obtain the equations of the linear theory of elasticity

$$\partial h_{11}/\partial t = \partial u/\partial x,\ \partial h_{22}/\partial t = \partial v/\partial y,\ \partial h_{33}/\partial t = 0,\ \partial h_{12}/\partial t = (\partial u/\partial y + \partial v/\partial x)/2.$$

The regions of elasticity are separated by the inequalities

$$h = \frac{1}{\sqrt{2}}[(h_{11} - h_{22})^2 + (h_{22} - h_{33})^2 + (h_{33} - h_{11})^2 + 6h_{12}h_{21}]^{1/2} < h_*$$
$$\text{or}\ \frac{d}{dt}\left[\sum_{i=1}^{3}\left(\sigma_{ii} - \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3}\right)^2 + 2\sigma_{12}\sigma_{21}\right] < 0 \quad (L = 0).$$

The parameter $L (0 \leq L \leq 1)$ characterizes the hardening of the medium with plastic deformations. In this case, system (2) is written in the form

$$\begin{gathered}
\rho_0 \partial u/\partial t = \partial\sigma_{11}/\partial x + \partial\sigma_{12}/\partial y,\ \rho_0 \partial v/\partial t = \partial\sigma_{21}/\partial x + \partial\sigma_{22}/\partial y, \\
\frac{\partial h_{11}}{\partial t} = \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{1}{2}\sin^2 2\varphi\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + \frac{1}{2}\sin 2\varphi\cos 2\varphi\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \\
\frac{\partial h_{22}}{\partial t} = \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \frac{1}{2}\sin^2 2\varphi\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) - \frac{1}{2}\sin 2\varphi\cos 2\varphi\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),
\end{gathered} \tag{3}$$

$$\frac{\partial h_{33}}{\partial t}=\frac{1}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right),\quad \frac{\partial h_{12}}{\partial t}=\frac{1}{2}\sin 2\varphi\cos 2\varphi\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)+\frac{1}{2}\cos^2 2\varphi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),$$

where $\sigma_{ij} = \lambda(h_{11} + h_{22} + h_{33})\delta_{ij} + 2\mu h_{ij}$.

If we consider the steady-state system arising from (3), i.e., if we delete the derivatives with respect to t, we obtain the system

$$\partial\sigma_{11}/\partial x + \partial\sigma_{12}/\partial y = 0,\ \partial\sigma_{21}/\partial x + \partial\sigma_{22}/\partial y = 0,\ \partial u/\partial x + \partial v/\partial y = 0,$$
$$\sin 2\varphi(\partial u/\partial x - \partial v/\partial y) + \cos 2\varphi(\partial u/\partial y + \partial v/\partial x) = 0,$$

if it is supplemented by the relationships

$$\sigma_{11} = \sigma_1\cos^2\varphi + \sigma_2\sin^2\varphi,\ \sigma_{22} = \sigma_1\sin^2\varphi + \sigma_2\cos^2\varphi,$$
$$\sigma_{12} = (\sigma_2 - \sigma_1)\sin\varphi\cos\varphi,\ (\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2 = 4k^2,$$

where k is the yield point, we obtain the classical equations of the theory of plasticity [5]. We note that a partial case of system (3) was proposed in [3].

We write the equation of the normals to the characteristics for system (3). For this purpose, we rewrite the equations for the velocities in the form

$$\frac{\partial u}{\partial t}=\frac{\lambda+2\mu}{\rho_0}\frac{\partial h_{11}}{\partial x}+\frac{\lambda}{\rho_0}\frac{\partial h_{22}}{\partial x}+\frac{\lambda}{\rho_0}\frac{\partial h_{33}}{\partial x}+\frac{2\mu}{\rho_0}\frac{\partial h_{12}}{\partial y},$$
$$\frac{\partial v}{\partial t}=\frac{2\mu}{\rho_0}\frac{\partial h_{12}}{\partial x}+\frac{\lambda}{\rho_0}\frac{\partial h_{11}}{\partial y}+\frac{\lambda+2\mu}{\rho_0}\frac{\partial h_{22}}{\partial y}+\frac{\lambda}{\rho_0}\frac{\partial h_{33}}{\partial y},$$

while the equations for h_{ij} are left without change.

For calculation of the characteristics, it is convenient to use a method, used, for example in [1], which consists in the following: we differentiate the equations for u and v once again with respect to t, and substitute into them the equations for h_{ij}, differentiated, as necessary, with respect to x or y. If we denote by (τ, ξ_1, ξ_2) the vector of the normal to the characteristic surface, then, the equation of the characteristics of the normals has the form

$$\det(\tau^2 I - \Lambda) = 0, \tag{4}$$

where

$$\Lambda=\begin{pmatrix}\Lambda_{11} & \Lambda_{12}\\ \Lambda_{21} & \Lambda_{22}\end{pmatrix};$$
$$\Lambda_{11}=\frac{1}{\rho_0}(K+\mu\sin^2 2\varphi)\xi_1^2+\frac{\mu}{\rho_0}\sin 4\varphi\xi_1\xi_2+\frac{\mu}{\rho_0}\cos^2 2\varphi\xi_2^2;$$
$$\Lambda_{22}=\frac{\mu}{\rho_0}\cos^2 2\varphi\xi_1^2-\frac{\mu}{\rho_0}\sin 4\varphi\xi_1\xi_2+\frac{1}{\rho_0}(K+\mu\sin^2 2\varphi)\xi_2^2;$$
$$\Lambda_{12}=\Lambda_{21}=\frac{\mu}{2\rho_0}\sin 4\varphi\xi_1^2+\frac{1}{\rho_0}(K+\mu\cos 4\varphi)\xi_1\xi_2-\frac{\mu}{2\rho_0}\sin 4\varphi\xi_2^2$$

$(K = \lambda + 2\mu/3)$. It can be shown that the matrix Λ is negatively defined. In actual fact, with $\xi_1^2 + \xi_2^2 \neq 0$

$$\Lambda_{11}=\frac{1}{\rho_0}\left[K\xi_1^2+\mu(\sin 2\varphi\xi_1+\cos 2\varphi\xi_2)^2\right]>0,$$
$$\Lambda_{22}=\frac{1}{\rho_0}\left[\mu(\cos 2\varphi\xi_1-\sin 2\varphi\xi_2)^2+K\xi_2^2\right]>0,$$
$$\det\Lambda=\frac{K\mu}{\rho_0^2}[\cos^2 2\varphi\xi_1^4-2\sin 4\varphi\xi_1^3\xi_2+2(\sin^2 2\varphi-\cos 4\varphi)\xi_1^2\xi_2^2$$
$$+2\sin 4\varphi\xi_1\xi_2^3+\cos^2 2\varphi\xi_2^4]=\frac{K\mu}{\rho_0^2}[2\sin 2\varphi\xi_1\xi_2-\cos 2\varphi(\xi_1^2-\xi_2^2)]^2\geqslant 0.$$

Consequently, Eq. (4) has real nonnegative roots τ^2.

The determinant of the matrix Λ reverts to zero with $2\sin 2\varphi\xi_1\xi_2 - \cos 2\varphi(\xi_1^2 - \xi_2^2) = 0$, i.e., with $\xi_2 = (1 \pm \sin 2\varphi)\xi_1$.

If we denote $\varphi = -(\theta + \pi/4)$, then, the equations of the surfaces, at which $\det \Lambda = 0$, have the form

$$\cos\theta\xi_1 + \sin\theta\xi_2 = 0,\ \sin\theta\xi_1 - \cos\theta\xi_2 = 0. \tag{5}$$

These equations coincide with the equations of the slip lines of the theory of plasticity [5]

$$dy = -\mathrm{ctg}\,\theta dx,\ dy = \mathrm{tg}\,\theta dx.$$

Thus, Eq. (4)

$$\tau^4 - (\Lambda_{11} + \Lambda_{22})\tau^2 + \det\Lambda = 0$$

with the conditions (5) has the roots

$$\tau^2 = 0,\ \tau^2 = \Lambda_{11} + \Lambda_{22} = \frac{K+\mu}{\rho_0}(\xi_1^2 + \xi_2^2).$$

Consequently, at the lines of slip, the velocity of the quasitransverse waves reverts to zero, while the velocity of the quasilongitudinal waves is equal to $(K + \mu)/\rho_0$.

We note that, with $L < 1$, the acoustical matrix corresponding to the system (2) does not have lines of degeneracy. The characteristic equation for this case, at the principal axes, is written out in [1].

We now write the acoustical matrix, describing the propagation of small perturbations in a plastic medium for the case of a plane stressed state [5]. The equations for this case are obtained from system (3), in which one of the equations for h_{11}, h_{22}, h_{33} is replaced by the algebraic equation

$$\sigma_{33} = \lambda(h_{11} + h_{22} + h_{33}) + 2\mu h_{33} = 0.$$

Omitting the calculations, we immediately write the result. The equation of the characteristics of the normals has the form

$$\det(\tau^2 I - \Lambda) = 0,$$

where

$$\begin{aligned}
\rho_0\Lambda_{11} &= \mu\left(\frac{2K}{\lambda+2\mu} + \sin^2 2\varphi\right)\xi_1^2 + \mu\sin 4\varphi\xi_1\xi_2 + \mu\cos^2 2\varphi\xi_2^2\\
&= \mu\frac{2K}{\lambda+2\mu}\xi_1^2 + \mu(\sin 2\varphi\xi_1 + \cos 2\varphi\xi_2)^2 > 0,\\
\rho_0\Lambda_{22} &= \mu\cos^2 2\varphi\xi_1^2 - \mu\sin 4\varphi\xi_1\xi_2 + \mu\left(\frac{2K}{\lambda+2\mu} + \sin^2 2\varphi\right)\xi_2^2\\
&= \mu(\cos 2\varphi\xi_1 + \sin 2\varphi\xi_2)^2 + \mu\frac{2K}{\lambda+2\mu}\xi_2^2 > 0,\\
\rho_0\Lambda_{12} = \rho_0\Lambda_{21} &= \frac{\mu}{2}\sin 4\varphi\xi_1^2 + \mu\left(\frac{2K}{\lambda+2\mu} + \cos 4\varphi\right)\xi_1\xi_2 - \frac{\mu}{2}\sin 4\varphi\xi_2^2,\\
\det\Lambda &= \frac{2\mu^2 K}{\rho_0^2(\lambda+2\mu)}\left[2\sin 2\varphi\xi_1\xi_2 - \cos 2\varphi(\xi_1^2 - \xi_2^2)\right]^2 \geqslant 0.
\end{aligned}$$

The lines of degeneration of the matrix Λ are found to be the same

$$\xi_2 = (1 \pm \sin 2\varphi)\xi_1.$$

This is in agreement with the fact that the lines of the slip for plane deformation and for a plane stressed state are exactly the same.

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